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Blowup of Solutions for a System of Doubly Nonlinear Degenerate Parabolic Equations with p-Laplacian

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Abstract

This paper is concerned with a system of doubly nonlinear degenerate parabolic equations with p-Laplacian. We prove that, under suitable conditions on the nonlinearity and certain initial datum, the lower bound for the blowup time is given if blowup does occur by using a modification of Levine's concavity method.

Keywords

Blowup of Solution, Doubly Nonlinear Parabolic Equations, Levine's Concavity Method

多重非线性退化的p-Laplacian抛物方程组解的爆破

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摘 要

本文研究了一类多重非线性退化的p-Laplacian抛物方程组解的爆破，利用修正的Levine凸性方法，在非线性项和初始条件的适当条件下，给出了解爆破时间的下界。

关键词

爆破，多重非线性抛物方程组，Levine凸性方法

1. 引言

本文研究如下非线性抛物方程组解的爆破性

$$\frac{\partial}{\partial t}\left(|u|^{m_1-2}u\right)-\Delta_{p_1}u=f_1(u,v), \tag{1.1}$$

$$\frac{\partial}{\partial t}\left(|v|^{m_2-2}v\right)-\Delta_{p_2}v=f_2(u,v), \tag{1.2}$$

$$u(x,t)=v(x,t)=0,\ \ x\in\partial\Omega,\ t\geq 0, \tag{1.3}$$

$$u(x,0)=u_0(x),\ \ v(x,0)=v_0(x),\ x\in\Omega, \tag{1.4}$$

其中 Ω 是 $R^n\ (n\geq 1)$ 上的有界区域且有光滑边界 $\partial\Omega$，$\Delta_{p_i}u=\operatorname{div}\left(|\nabla u|^{p_i-2}\nabla u\right)$，$2\leq p_1,p_2<+\infty, m_1,m_2>2$，$f_i(u,v), i=1,2$ 为以后给定的函数，$u_0(x), v_0(x)$ 为正的初始函数。

近几十年来，非线性抛物方程解的爆破问题引起人们的极大兴趣，考虑爆破性自然要研究解是否会爆破以及解的爆破时间 t^*，在这方面已经有大量的文献，如文献[1]-[5]。问题(1.1)~(1.4)可以描述诸多化学反应、热传导过程和种群动力学过程(见文献[6]-[17]及其参考文献)，它有多个非线性项，处理难度较大。对型如(1.1)的单个多重非线性抛物方程

$$\frac{\partial}{\partial t}\left(|u|^{m-2}u\right)-\Delta u=f(u), \tag{1.5}$$

已有许多结果，如文献[6]-[10]给出了方程(1.5)的初边值问题和初值问题局部和整体解的存在性，文献[11]-[14]则研究了其整体吸引子的存在性和正则性，类似方程(1.1)，(1.2)的方程组问题的整体吸引子的存在性和正则性也有一些研究[15]-[17]。但关于该类问题解的爆破性研究则相对较少，Iami 和 Mochizuki [18] 给出了 Neumann 初边值条件下方程(1.5)的爆破条件，Levine [19]-[21]则用凸性方法证明了单个方程(1.1)及其等价的如下问题解的爆破性

$$\frac{\partial u}{\partial t}-\Delta\varphi(u)=f(u).$$

Korpusov 和 Sveshnikov [22] [23]及 Polat [24]则对如下方程的初边值问题

$$\left(u-\Delta u+|u|^{k-2}u\right)_t-\Delta u=f(u)$$

在负初始能量时得到解的爆破条件。Wang 和 Ge [25]及其参考文献则利用比较原理讨论了非线性边界时方程(1.1)，(1.2)中非线性项为 $u^\alpha v^\beta$ 时解的爆破问题。

本文利用修正的 Levine 凸性方法讨论问题(1.1)~(1.4)解的爆破条件，推广了文献[22]-[25]的结果。在第二节将给出一些假设和基本引理，第三节给出主要结果和证明。

2. 假设和引理

本文所用符号均同文献[6]，记 $W_0^{1,p}(\Omega)$ 和 $L^m(\Omega)$ 为通常的 Soblev 空间，其范数分别记为 $\|.\|_{W_0^{1,p}} = \|\nabla .\|_p$ 和 $\|.\|_m$，特别是当 $m=2$ 时，记 $\|.\|_m = \|.\|$。

本文始终假设 $2 \le p_1, p_2 < +\infty, m_1, m_2 > 2$。关于非线性项 $f_i(u,v), i=1,2$ 的假设如下：

(A1) $f_i(u,v) \in C^1(R^2), i=1,2$，存在函数 $F(u,v) \in C^2(R^2)$ 使得

$$f_1(u,v) = \frac{\partial F(u,v)}{\partial u},\quad f_2(u,v) = \frac{\partial F(u,v)}{\partial v},$$

且存在常数 $\beta_0 > 2, \beta_1 > 0$ 使得

$$\beta_1\left(|u|^{p+1} + |v|^{p+1}\right) \le F(u,v) \le \frac{1}{\beta_0}\left(uf_1(u,v) + vf_2(u,v)\right),$$

其中，$p > 0$ 当 $n = 1, 2$ 时，$0 < p < \frac{n+2}{n-2}$ 当 $n > 2$ 时。

注：满足条件(A1)的函数是存在的。事实上，一个典型的例子是取

$$F(u,v) = a|u+v|^{p+1} + 2b|uv|^{\frac{p+1}{2}}$$

且 $f_1(u,v) = \frac{\partial F(u,v)}{\partial u}, f_2(u,v) = \frac{\partial F(u,v)}{\partial v}$，即

$$f_1(u,v) = (p+1)\left(a|u+v|^{p-1}(u+v) + b|u|^{\frac{p-3}{2}}u|v|^{\frac{p+1}{2}}\right),$$

$$f_2(u,v) = (p+1)\left(a|u+v|^{p-1}(u+v) + b|v|^{\frac{p-3}{2}}v|u|^{\frac{p+1}{2}}\right),$$

这时，$(p+1)F(u,v) = uf_1(u,v) + vf_2(u,v)$，其中 $a > 0, b > 0$，$p \ge 1, \beta_0 = p+1$。该例的详细情况可见文献[26]。

利用 Galerkin 方法，结合单调性理论和紧性方法[2]，类似文献[27]可得问题(1.1)~(1.4)解的局部存在性。

定理 2.1. 假设条件(A1)成立且 $\min(p_1, p_2) > n(p+1)/(n + \max(m_1, m_2))$，$\max(m_1, m_2) \le p+1$，$u_0 \in W_0^{1,p_1}, v_0 \in W_0^{1,p_2}$，则问题(1.1)~(1.4)存在弱解 (u,v)，即，存在 $T > 0$ 使得

$$u \in L^\infty\left(0,T;W_0^{1,p_1}\right),\quad v \in L^\infty\left(0,T;W_0^{1,p_2}\right),\quad \left(|u|^{\frac{m_1}{2}}\right)_t, \left(|v|^{\frac{m_2}{2}}\right)_t \in L^2\left((0,T)\times\Omega\right),$$

且对任意 $\varphi(t) \in D(0,T)$ 和任意分别的 $\phi(x) \in W_0^{1,p_1}$ (或 W_0^{1,p_2})成立：

$$\int_0^T \left[\int_\Omega \left[\frac{2(m_1-1)}{m_1}|u|^{m_1/2-2}u\left(|u|^{m_1/2}\right)_t\phi + |\nabla u|^{p_1-2}\nabla u \nabla\phi - f_1(u,v)\phi\right]\mathrm{d}x\varphi(t)\right]\mathrm{d}t = 0,$$

$$\int_0^T \left[\int_\Omega \left[\frac{2(m_1-1)}{m_1}|v|^{m_1/2-2}v\left(|v|^{m_1/2}\right)_t\phi + |\nabla v|^{p_2-2}\nabla v \nabla\phi - f_2(u,v)\phi\right]\mathrm{d}x\varphi(t)\right]\mathrm{d}t = 0,$$

以及 $u(x,0) = u_0 \in W_0^{1,p_1}, v(x,0) = v_0 \in W_0^{1,p_2}$。

下面给出本文的基本引理。

引理 2.2. [5] [27] 设 $H(t)$ 是 R 上非负二次连续可导函数且满足不等式

$$\Phi''(t)\Phi(t) - \alpha\left[\Phi'(t)\right]^2 + \beta\Phi(t) \ge 0$$

其中 $\alpha>1,\beta>0$ 为常数。若 $\Phi(0)>0$，$\Phi'(0)>0,\left[\Phi'(0)\right]^2>\dfrac{2\beta}{2\alpha-1}\Phi(0)$，则必存在时刻 $T<T_1=A^{-1}\Phi^{1-\alpha}(0)$，使当 $t\to T^-$ 时有 $\Phi(t)\to+\infty$，其中

$$A^2=(\alpha-1)^2\Phi^{-2\alpha}(0)\left[\left(\Phi'(0)\right)^2-\frac{2\beta}{2\alpha-1}\Phi(0)\right]>0.$$

3. 主要结果及证明

首先引入泛函

$$\Phi(t)=\int_0^t\left(\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\right)\mathrm{d}s+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}, \tag{3.1}$$

$$J(t)=\int_0^t\int_\Omega\left[(m_1-1)|u|^{m_1-2}u_t^2+(m_2-1)|v|^{m_2-2}v_t^2\right]\mathrm{d}x\mathrm{d}s+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}, \tag{3.2}$$

$$E(t)=\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}+\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}-\int_\Omega F(u,v)\mathrm{d}x, \tag{3.3}$$

$$E(0)=\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}+\frac{1}{p_1}\|\nabla u_0\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v_0\|_{p_2}^{p_2}-\int_\Omega F(u_0,v_0)\mathrm{d}x. \tag{3.4}$$

现给出主要引理。

引理 3.1. 对任意 $t\in[0,T)$，下面不等式成立

$$\left[\Phi'(t)\right]^2\le(m_1+m_2)\Phi(t)J(t). \tag{3.5}$$

证明 注意到

$$\begin{aligned}\Phi'(t)&=\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\\&=\int_0^t\frac{\mathrm{d}}{\mathrm{d}s}\left[\frac{m_1-1}{m_1}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v\|_{m_2}^{m_2}\right]\mathrm{d}s+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2},\end{aligned} \tag{3.6}$$

注意到由 Holder 不等式得下列不等式

$$\left|\int_0^t\frac{m_1-1}{m_1}\frac{\mathrm{d}}{\mathrm{d}s}\|u\|_{m_1}^{m_1}\mathrm{d}s\right|\le(m_1-1)\int_0^t\int_\Omega|u|^{\frac{m_1}{2}}\left(|u|^{\frac{m_1-2}{2}}u_t\right)\mathrm{d}x\mathrm{d}s\le(m_1-1)\left(\int_0^t\|u\|_{m_1}^{m_1}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|u|^{m_1-2}|u_t|^2\mathrm{d}x\mathrm{d}s\right)^{\frac12}, \tag{3.7}$$

$$\left|\int_0^t\frac{m_2-1}{m_2}\frac{\mathrm{d}}{\mathrm{d}s}\|v\|_{m2}^{m_2}\mathrm{d}s\right|\le(m_2-1)\left(\int_0^t\|v\|_{m_2}^{m_2}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|v|^{m_2-2}|v_t|^2\mathrm{d}x\mathrm{d}s\right)^{\frac12}. \tag{3.8}$$

考虑到(3.7)~(3.8)，则由(3.6)得

$$\begin{aligned}\Phi'(t)\le&(m_1-1)\left(\int_0^t\|u\|_{m_1}^{m_1}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|u|^{m_1-2}|u_t|^2\mathrm{d}x\mathrm{d}s\right)^{\frac12}\\&+(m_2-1)\left(\int_0^t\|v\|_{m_2}^{m_2}\mathrm{d}s\right)^{\frac12}\left(\int_0^t\int_\Omega|v|^{m_2-2}|v_t|^2\mathrm{d}x\mathrm{d}s\right)^{\frac12}\\&+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2},\end{aligned}$$

再利用不等式

$$\left(\sum_{i=1}^{k} a_i b_i\right)^2 \le \left(\sum_{i=1}^{k} a_i^2\right)\left(\sum_{i=1}^{k} b_i^2\right),$$

得

$$\begin{aligned}(\Phi'(t))^2 \le& \left[\int_0^t\left((m_1-1)\|u\|_{m_1}^{m_1}+(m_2-1)\|v\|_{m_2}^{m_2}\right)\mathrm{d}s+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\right]\\ &\cdot\left[\int_0^t\int_\Omega\left((m_1-1)|u|^{m_1-2}|u_t|^2+(m_2-1)|v|^{m_2-2}|v_t|^2\right)\mathrm{d}x\mathrm{d}s+\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\right]\\ &\le (m_1+m_2)\left[\int_0^t\left(\frac{m_1-1}{m_1+m_2}\|u\|_{m_1}^{m_1}+\frac{m_2-1}{m_1+m_2}\|v\|_{m_2}^{m_2}\right)\mathrm{d}s+\frac{m_1-1}{m_1(m_1+m_2)}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2(m_1+m_2)}\|v_0\|_{m_2}^{m_2}\right]J\\ &\le (m_1+m_2)\Phi J.\end{aligned}$$

于是，引理得证。

下面，给出主要定理。

定理 3.2. 设定理 2.1 的条件成立，且

$$\left[4(m_1+m_2)-5\right]\left[\frac{m_1-1}{m_1}\|u_0\|_{m_1}^{m_1}+\frac{m_2-1}{m_2}\|v_0\|_{m_2}^{m_2}\right]\ge 4\left[\frac{1}{p_1}\|\nabla u_0\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v_0\|_{p_2}^{p_2}-\int_\Omega F(u_0,v_0)\mathrm{d}x\right], \tag{3.9}$$

则问题(1.1)~(1.4)的弱解 (u,v) 必在某有限时刻 $T<T_1=A^{-1}\Phi^{1-\alpha}(0)$ 爆破，即

$$\limsup_{t\to T}\left(\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}\right)=+\infty.$$

证明 注意到

$$\frac{m_1-1}{m_1}\frac{\mathrm{d}}{\mathrm{d}t}|u|^{m_1}=(m_1-1)u|u|^{m_1-2}u_t=u\frac{\mathrm{d}}{\mathrm{d}t}\left(u|u|^{m_1-2}\right),$$

$$\frac{m_2-1}{m_2}\frac{\mathrm{d}}{\mathrm{d}t}|v|^{m_2}=(m_2-1)v|v|^{m_2-2}v_t=v\frac{\mathrm{d}}{\mathrm{d}t}\left(v|v|^{m_2-2}\right),$$

由(3.1)，得

$$\Phi''(t)=\frac{m_1-1}{m_1}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^{m_1}+\frac{m_2-1}{m_2}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^{m_2}=\int_\Omega\left(u\frac{\mathrm{d}}{\mathrm{d}t}\left(u|u|^{m_1-2}\right)+v\frac{\mathrm{d}}{\mathrm{d}t}\left(v|v|^{m_2-2}\right)\right)\mathrm{d}x.$$

把(1.1)，(1.2)代入得

$$\Phi''(t)+\|\nabla u\|_{p_1}^{p_1}+\|\nabla v\|_{p_2}^{p_2}=\int_\Omega\left(f_1(u,v)u+f_2(u,v)v\right)\mathrm{d}x, \tag{3.10}$$

利用

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(u|u|^{m_1-2}\right)=(m_1-1)|u|^{m_1-2}u_t,\quad \frac{\mathrm{d}}{\mathrm{d}t}\left(v|v|^{m_2-2}\right)=(m_2-1)|v|^{m_2-2}v_t,$$

由(3.2)及方程(1.1)，(1.2)得

$$\begin{aligned}J'(t)&=(m_1-1)\int_\Omega|u|^{m_1-2}|u_t|^2\mathrm{d}x+(m_2-1)\int_\Omega|v|^{m_2-2}|v_t|^2\mathrm{d}x\\ &=\int_\Omega u_t\left(|u|^{m_1-2}u\right)_t\mathrm{d}x+\int_\Omega v_t\left(|v|^{m_2-2}v\right)_t\mathrm{d}x\\ &=-\int_\Omega|\nabla u|^{p_1-2}\nabla u\nabla u_t\mathrm{d}x-\int_\Omega|\nabla v|^{p_2-2}\nabla v\nabla v_t\mathrm{d}x\\ &\quad+\int_\Omega\left(f_1(u,v)u_t+f_2(u,v)v_t\right)\mathrm{d}x,\end{aligned}$$

即

$$J'(t)+\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}\right)=\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega F(u,v)\mathrm{d}x, \tag{3.11}$$

(3.11)关于 t 积分得

$$J(t)+\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}-E(0)=\int_\Omega F(u,v)\mathrm{d}x, \tag{3.12}$$

再利用条件(A1)得

$$\beta_0\left(J(t)+\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}-E(0)\right)\le\int_\Omega\left(f_1(u,v)u+f_2(u,v)v\right)\mathrm{d}x, \tag{3.13}$$

(3.10)结合(3.13)，并用到 $\beta_0>2$, 得

$$\begin{aligned}\Phi''(t)+\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}&\ge 2\left(J(t)+\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}-E(0)\right)\\&\ge 2J(t)+\|\nabla u\|^2+\|\nabla v\|^2-2E(0),\end{aligned}$$

即

$$\Phi''(t)-2J(t)+2E(0)\ge 0,, \tag{3.14}$$

注意到 $\Phi(t)\ge 0$ 得

$$\Phi''(t)\Phi(t)-2J(t)\Phi(t)+2E(0)\Phi(t)\ge 0, \tag{3.15}$$

利用引理 3.1，得

$$\Phi''(t)\Phi(t)-\alpha\left[\Phi'(t)\right]^2+\beta\Phi(t)\ge 0,$$

其中 $\alpha=2(m_1+m_2),\beta=2E(0)$。

如果 $E(0)>0$，由(3.1)，(3.6)和(3.9)知 $\Phi(0)>0$， $\Phi'(0)>0,\left[\Phi'(0)\right]^2>\frac{2\beta}{2\alpha-1}\Phi(0)$，于是，由引理 2.2 得结论。如果 $E(0)\le 0$，取 $\beta=0$，则(3.15)变为

$$\Phi(t)\Phi(t)-\alpha\left[\Phi'(t)\right]^2\ge 0, \tag{3.16}$$

于是，由(3.16)和标准的凸性引理得结论。综合两种情况即得 $\limsup_{t\to T}\left(\frac{1}{p_1}\|\nabla u\|_{p_1}^{p_1}+\frac{1}{p_2}\|\nabla v\|_{p_2}^{p_2}\right)=+\infty$。

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